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UNSTEADY CAVITATIONAL FLOW OVER A DISK*

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A method is proposed for calcuating the unsteady flow over bodies without using simplified assumptions about the form of these and the cavity behind them, the velocity of motion of the body or the pressure of the fluid. Calculations are given of axisymmetric unsteady cavities behind the disk for various modes of motions and changes of pressure in the cavity.

Previously calculations of unsteady cavitational flows were carried out only with substantial simplifications both in plane /1/ and in axisymmetric flows (in the latter case only for thin cavities /2—5/ or infinite cavities /6, 7/.

1. A disk moves in a medium at rest with velocity V. The fluid is assumed to be perfectly wightless and incompressible, and the flow is assumed to be irrotational. Outside the body and its cavity we can introduce a scalar velocity potential Φ in the form of a harmonic function satisfying the slip condition

$$\frac{\partial\Phi}{\partial N} - (N, V_S) = 0 \qquad (1.1)$$

on $S = S_b \cup S_c \cup S_j$. Here N is the unit vector of the outside normal to the surface S consisting of the wetted disk surface S_b, the surface of the cavity S_c, and the surface S_j closing the cavity of the imaginary disk /8/. V_S is velocity of motion of the point on S: $V_S|_{S_b} = V$.

The presence of a fictitious body enables us to confine ourselves to studying the cavity reaction to a change in the flow around the body, without considering the pulsation of the cavity tail which is observable even in steady flow.

The Cauchy-Lagrange condition of constant pressure is satisfied on S_c, i.e.

$$p = \rho\frac{\partial\Phi}{\partial t} - \rho\frac{v^2}{2}$$

where ρ is the fluid density, v is its velocity in the absolute system of coordinates, and p is the pressure difference in the cavity and the unperturbed flow.

It is convenient to use, as in /9/, a coordinate system attached to the disk and to make V the unit of velocity. Denoting by U the fluid velocity in that coordinate system, the condition of constant pressure in the cavity can be written in the form

$$\sigma \equiv \frac{2p}{\rho V^2} = U^2 - 1 \alpha\eta^2\Phi + 2\eta\frac{\partial\Phi}{\partial\tau} \qquad (1.2)$$
$$\alpha = DV_0^{-2}dV/dt, \quad \eta = V_0V^{-1}, \quad V_0 = V(0), \quad \tau = tV_0D^{-1}$$

(where τ is dimensionless time, D is the disk diameter, σ is the cavitation coefficient, and the velocity potential is taken relative to the product VD).

Let p, V be given functions of τ. If the quasistationary approach is used in calculations of cavitation, i.e. the last two terms in (1.2) are neglected, it is convenient to specify the sequence of cavity lengths

$$\{L_1, L_2, \ldots\} \qquad (1.3)$$

and to obtain the respective sequence $\{\sigma_1, \sigma_2, \ldots\}$ from the solution of the problem. The complete Eq.(1.2) contains three parameters, each of which affects the cavity length σ, α, η, which all depend on τ. Hence it is convenient in calculations of unsteady axisymmetric cavities to specify the sequence of values of (1.3), and use these to find the respective $\{\tau_1, \tau_2, \ldots\}$. The sequence (1.3) cannot be entirely arbitrary, since the sequence $\{\tau_1, \tau_2, \ldots\}$ must retain monotonous.

The determination of the form of each cavity length is a non-linear problem, solved by the method of successive approximations (similar to those used in /8, 10/). In each approximation the Neuman problem is solved for the Laplace equation for Φ with condition (1.1) outside S; then the discrepancy in (1.2) is checked, and if it exceeds an a priori specified positive number, it is corrected using the linearized conditions (1.1) and (1.2)

$$\frac{\partial\varphi}{\partial N} + \frac{\partial Uh}{\partial T} + \eta\frac{h}{\tau_*} = 0 \qquad (1.4)$$

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$$U\frac{\partial\varphi}{\partial\lambda}-\varphi w+\left(\frac{\dot\sigma}{2}+\frac{\Phi_-\dot\eta}{\tau_*}-\Phi_0\dot w\right)\tau'=\frac{U^2-1-\sigma}{2}+\Phi_0 w-\eta\frac{\Phi_-}{\tau_*},\quad w=\alpha\eta^2+\frac{\eta}{\tau_*}. \tag{1.5}$$

Here λ, T are the tangents to the equipotential streamlines $\tau'=\tau_+-\tau_0$, $\tau_*=\tau_0-\tau_-$; the quantities explicitly dependent on time with zero subscript relate to τ_0, and with a minus subscript to τ_-; the derivatives of these quantities with respect to τ are denoted by dots; h is the distance between the required and the known surfaces measured along N; and φ is the perturbation of Φ. The functionals Φ, φ are the potentials of a simple layer.

The unkown function in (1.5) is the density q of the potential φ and the number τ'. Since when $\tau=\tau_-$ (the time point in previous calculation) the flow over the body with a cavity has already been determined, the calculation of Φ_- does not in principle present difficulties. It was assumed in deriving (1.5) that the required time τ_+ (corresponding to the cavity length L_K) is not much different from its approximate value τ_0, i.e. $|\tau'/\tau_*|\ll 1$. For each length L_K the value of τ_0 in the initial approximation is selected to minimize the maximum value of the discrepancy in (1.2), without making more precise the form of S_c; the discrepancy in the initial approximation is treated as a function of only one parameter τ_0. In subsequent approximations τ' is added to τ_0.

The singular integral Eq.(1.5) is adjusted by using the Keldysh-Sedov formula /11/, and from the additional condition required for its use we determine τ', and for the construction of a new approximation to the boundary of cavity of length L_K (1.4) we use (1.4) with condition $h(0)=0$ on the cavitating body.

Compared with the quasisteady approach the volume of calculations increases due both to the need for each $\tau>0$ to recall not less than two fields Φ, and due to the need begin the calculations from any τ, and that to determine Φ one must begin the calculations from some steady cavitational flow (when $\Phi_0\equiv\Phi_-$).

2. Some results of calculations of the flow over a disk are presented below for various laws of variation of $p(\tau)$ and $V(\tau)$.

$$\alpha(\tau)=\beta,\ \eta(\tau)=(1+\beta\tau)^{-1},\ \sigma(\tau)=\sigma(0)\,\eta^2 \tag{2.1}$$

$$\alpha(\tau)=0,\ \eta(\tau)=1,\ \sigma(\tau)=\sigma(0)\,(1+\beta\tau)^{-2} \tag{2.2}$$

$$\alpha(\tau)=0,\ \eta(\tau)=1,\ \sigma(\tau)=\sigma(0)+A\sin(\omega\tau) \tag{2.3}$$

where β, A, ω are constants. For the same β the motion (2.1) of σ varies only on account of acceleration, and in (2.2) the variation is due only to the change of pressure at constant velocity V. In all calculations $\sigma(0)=0.136$ was assumed.

The dependence of the length and its width B of the cavity and of the coefficient of drag C_x on σ is shown in Fig.1. The solid curves *1*, *2* and *3* relate to braking of the disk in the mode (2.1) for $\beta=-0.002;\ -0.02;\ -0.1$. Curves 0 correspond to calculations using the hypothesis of pseudo-steadiness.

The dash lines show the results of mode (2.2) for $\beta=-0.02$. It is seen that the cavities differ little from steady for mode (2.2) (for the same σ). The small difference in C_x is due to the cavity tail having a lesser effect on the pressure distribution in the cavity. The form of the curves $L(\sigma)$ and $B(\sigma)$ shown in Fig.1 corresponds qualitatively to the linear theory /4,5/, and for the example calculated in Nesteruk's dissertation for $L/D>10$ and $\beta=-0.1$ some small cavity enlargement occurred at the beginning of the motion (comparison of cavities behind various bodies for small σ is admissible, since the form of the cavities depend only slightly on the form of the cavitating body).

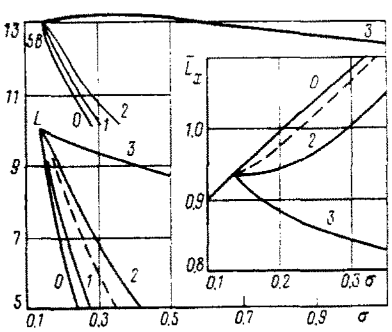

Fig.1

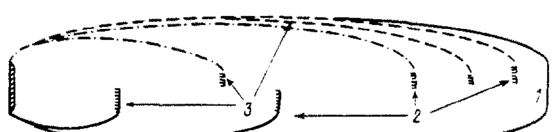

Fig.2

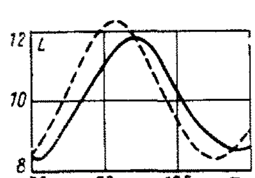

Fig.3

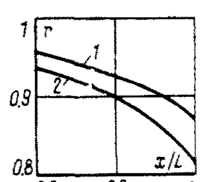

Fig.4

The form of cavities for mode (2.1) is shown in Fig.2 for the same σ but various β. The solid lines relate to steady cavities, the dash lines for $\beta = -0.1$, and the dash-dot lines for $\beta = -0.02$. The cavities at $\sigma = 0.136, 0.25, 0.5$ are denoted by numerals *1*, *2*, and *3*.

In Fig.3 the dependence $L(\tau)$ for $A = 0.023$; $\omega = 0.063$ for mode (2.3) are shown by solid lines, and the dash lines relate to calculations of the steady state for the same $\sigma(\tau)$. The distribution of $r = \frac{U^2}{1+\sigma}$ along the cavity for mode (2.1) for $\beta = -0.02$ are given in Fig.4; curves *1* and *2* relate to $\tau = 17$ and 23.

The convenience of judging the degree of unsteadiness of cavities using the value of r is obvious.

We may add that owing to the weak dependence of the form of the cavity on the shape of the cavitating body results of unsteady cavitational flows given here can be generalized considerably.

REFERENCES

1. KUZNTSOV A.V., Unsteady Perturbations of Fluid Flow with Free Boundaries. Kazan', Izd. Kazansk. Univ., 1975.
2. LOGVINOVICH G.V., Hydrodynamics of Flows with Free Boundaries. Kiev, Nauk. Dumka, 1969.
3. KONSTANTINOV G.A. and YAKIMOV YU.L., A numerical method of solving unsteady axisymmetric problems of the hydromechanics of an ideal fluid with free boundaries. Izv. AN SSSR, MZhG 4, 1969.
4. SEREBRYAKOV V.V., Calculations of Thin Axisymmetric Cavities in Unsteady Flow for Several Characteristic Cases of Flow. Hydromechanics. Collection of articles. Kiev, Naukova Dumka, 44, 1981.
5. NESTERUK G.I., On the form of a thin axisymmetric steady cavity. Izv. AN SSSR, MZhG, 4, 1980.
6. SHIMKOVICH D.M., The impact of a solid of revolution in a flow with separation. Izv. AN SSSR, MZhG, 6, 1980.
7. SHIFRIN E.G. and SHUBNIKOV G.V., A numerical method of solving problems of unsteady potential flow of fluid with moving boundaries. Zh. Vychisl. Matem. i Matem Fiz., 22, 1, 1982.
8. IVANOV A.N., Hydromechanics of Developed Cavitational Flows. Leningrad Sudostroenie, 1980.
9. POLYAKHOV N.N., Theory of Unsteady Motions of a Lifting Surface. Leningrad, Izd. LGU, 1960.
10. AMROMIN E.L., BUSHKOVSKII V.A. and DIANOV D.I., The development of cavitation behind a disk in a vertical tube. Izv. AN SSSR, MZhG, 1983.
11. SEDOV L.I., Plane Problems of Hydrodynamics and Aerodynamics. Moscow, Nauka, 1966.

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THE STEADY SPECTRA OF PARTICLES IN DISPERSIBLE SYSTEMS WITH COAGULATION AND FRAGMENTATION*

V.N. PISKUNOV

The formation of a steady dimensional distribution of particles (particle spectra) is dispersible systems with coagulation and fragmentation is considered. The relation between versions of the kinetic equation that defines these processes is traced. An analytical solution is obtained for the parametric set of coagulation coefficients and the velocities of paired fragmentation. The steady spectrum of particles is investigated in the case when the fragmentation is of the multiple type.

The kinetic equation of coagulation with fragmentation in the case when the rate of particle supply to the system to compensate for the fragmented particles is linear with respect to their concentration was first formulated in /1/. The fragmentation process can stabilize a coagulating dispersed system, and result in the formation of steady spectra. Some analytical results on the behaviour of systems with coagulation and fragmentation were obtained in /2—5/.

1. The variation with time t of the particle spectrum in three-dimensionally homogeneous systems with coagulation and fragmentation is defined by the kinetic equation

$$\partial c(g,t)/\partial t = S(c;g,t) + Q(c;g,t) \tag{1.1}$$

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